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Compressible Flow in a Hovercraft Air Cushion

A. Pozzi,* F. Manzo,† and P. Luchini†
University of Naples, 80125 Naples, Italy

A solution of the compressible fluid-dynamic equations describing the two-dimensional, inviscid, subsonic fluid-dynamic field in the peripheral jets of a hovercraft at zero forward speed is presented. The analysis is made in the hodograph plane in which the motion equation is linear and enables one to write the solution of the problem as a sum of elementary separable solutions of the equation. The stream function is expressed as a Fourier series whose coefficients can be calculated numerically with ease. The comparison of the results with those of a different representation, valid only in the incompressible case, shows that the two solutions coincide. The calculation of quantities of interest for several values of the parameters characterizing the problem enabled us to evaluate the influence of the compressibility that, for high subsonic Mach numbers of the jet, is at most on the order of magnitude of 20%.

Nomenclature

a = sound velocity
C_j = momentum jet coefficient, Eq. (30)
C_L = lift coefficient
C_Q = mass flux coefficient
C_w = power coefficient
h = height from ground
J = momentum of the jet
l_T = total width of machine
L_T = length of machine
M = V/a Mach number
p_c = air cushion overpressure, $W/L_T l_T$
t = width of the jet
V = modulus of velocity
W = weight of machine
x,y = Cartesian axes of reference
α = jet exit angle
γ = ratio of specific heats
θ = angle of the velocity vector with x axis
ρ = density
φ = velocity potential
ψ = stream function

Subscripts

i = conditions on the inner streamline
o = ambient outer conditions
s = stagnation conditions

Superscript

$+$ = nondimensional quantities referred to corresponding stagnation values

Introduction

THE study of machines (see Fig. 1) that, close to the ground, generate a base pressure under them greater than the ambient one by means of peripheral jets, was performed 40 years ago. The analysis was made in the incompressible regime because the Mach number of the jet required in typical operation (lift coefficient on the order of magnitude 1 and a ratio between distance from ground and total width of machine less than 0.1) was less than 0.3. With the construction of ever heavier and more powerful hovercraft, it became more and more important to include compressibility effects in the description of the fluid-dynamic field underneath the machine; for this purpose several approximate studies of compressible peripheral jets[1-3] were performed. There is today a renewed interest in high-performance ground-effect machines.[4]

The analyses of Ref. 4 lead to the following conclusions. Denoting by h, L_T, l_T, and W, height from ground, length, total width, and weight (expressed in tons) of machine, good performance of the hovercraft can be obtained if $h/L_T = 0.014\ W^{-1/3}$, $1 < W < 350$, and $1 \ll L_T/l_T < 6$; it follows that $0.002 < h/l_T < 0.084$. These data lead to a lift coefficient C_L on the order of magnitude 1, but in particular cases (motion over an irregular surface) values of h/l_T greater than 0.1 must be considered.

The velocity on the limiting streamline V_o is related to C_L by $V_o^2 = 2\ p_c/(\rho_o C_L)$, where ρ_o is the air density and $p_c = W/(L\ l_T)$ is the air cushion overpressure; good performance is obtained today when the values of p_c are in the range of 140 to 840 kg/m^2. Therefore, one has for V_o and the related Mach number M_o 40 m/s $< V_o < 130$ m/s and $0.12 < M_o < 0.38$. There is a trend to increase the value of p_c; in particular for operation on arctic tundras values on the order of 2400 kg/m^2 will be considered. The corresponding values of V_o and M_o are 220 m/s and 0.64.

It is therefore important to have exact solutions available that describe the compressible peripheral jet.

The first results regarding the incompressible flow in a hovercraft air cushion were obtained by means of simple

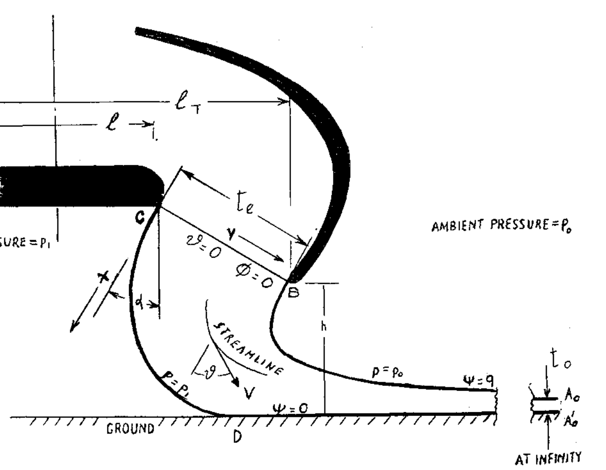

Fig. 1 Scheme of the peripheral jet.

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*Full Professor, Istituto di Gasdinamica, Facoltá di Ingegneeria, P.le Tecchio 80.

†Associate Professor, Istituto di Gasdinamica, Facoltá di Ingegneria, P.le Tecchio 80.

models, presented in reports of the U.S. Army and Navy.[5,6] The authors of early analyses neglected the jet's thickness for the purpose of the determination of lift. In this way they calculated the lift within a 20% error. An improvement of these theories was obtained by Pinnes[7] and Rethorst and Royce.[8] Strand[9] presented an exact solution of a model that assumes the fluid at rest in the region enclosed by the peripheral jets, using the conformal mapping technique. He determined the fluid-dynamic field in terms of transcendental functions and studied in detail the quantities of physical interest. Strand's technique cannot be extended to the compressible regime because the conformal mapping theory is applicable only to the solution of the Laplace equation. Approximate studies of the compressible peripheral jet, based on various simplifying assumptions, were conducted by Roche,[1] West,[2] and Hope-Gill.[3]

Our procedure, valid in both the incompressible and compressible regimes, starts from the equations of motion written in terms of the hodograph independent variables, V (modulus of the velocity vector $\mathbf{V}$) and θ (angle between $\mathbf{V}$ and a reference axis). In this formulation the equations are linear and the solution can be represented as a sum of products of functions $f_n(V)$ and $g_n(\theta)$. We find that $g_n(\theta)$ are sinusoidal functions of a suitable argument in both the incompressible and compressible cases. The functions $f_n(V)$ are very simple in the incompressible case ($f_n = V^n - V^{-n}$) but cannot be represented in terms of elementary functions in the compressible one.

A suitable organization of the procedure enables us to represent the solution of the problem in an exact way by finding a general expression for all of the coefficients we need for summing the elementary products that constitute the solution.

As is well known, Chaplygin's hodograph method yields the general solution of the problem of plane compressible inviscid flow in terms of hypergeometric functions. This method has often been applied to obtain approximate solutions (approximate because they do not exactly satisfy boundary conditions) once the corresponding incompressible solution is known. An example of this kind of approximation is given in Ref. 10 for a freejet. Our solution, instead, is calculated so as to satisfy the following boundary conditions exactly: issuing velocity perpendicular to the issuing cross section, constant velocity on the free surfaces, and zero normal velocity at the plane jet-ground interface.

Past Results

The first approaches to the problem of two-dimensional peripheral jets issuing from a hovering vehicle[5,6] only considered the equilibrium, across a jet without thickness, between centrifugal and pressure forces, thus finding

$$P_i - p_o = j(1 - \sin\alpha)/h \tag{1}$$

where p_i and p_o are the base and ambient pressure. Equation (1) shows that the base pressure increases without bounds as h vanishes. This theory was improved by Pinnes[7] and Rethorst and Royce,[8] who considered jets whose thickness t_e was small with respect to h. In this way those authors found that the base pressure never exceeds the jet's total head. They wrote the following simplified force balance equation in the radial direction in a polar coordinate system:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \rho\frac{U^2}{r} \tag{2}$$

where r is the radial distance, p is the pressure, U is the velocity, and ρ is the fluid density, assumed constant. Equation (2), together with the Bernoulli equation, gives the following more realistic expression for the base pressure:

$$(p_i - p_o)/(H - p_o) = 1 - (U_i/U_o)^2 \tag{3}$$

where H is the jet's total head, and U_i and U_o are the velocities on the inner and outer side of the jet, respectively.

Strand[9] presented an exact solution found by means of the conformal mapping technique. By introducing the complex potential $F = \varphi + i\psi$ and the complex velocity $w = \log(U_o/V) + i\theta$, where φ is the scalar potential, ψ is the stream function, and V and θ are the modulus and the angle (with respect to the x reference axis) of the velocity vector, he found

$$w = \frac{\pi/2 - \alpha}{K'}\int_0^Z \frac{\mathrm{d}t}{[(1-t^2)(1-k^2t^2)]^{1/2}} \tag{4}$$

where $Z = \cosh(\pi F/2q)$, $F = \varphi + i\psi$, $q = U_o t_o$, t_o is the jet width at infinity, $K(k)$ and $K'(k')$ are the complete elliptic integrals of the first kind with modulus k and k', respectively, with $k'^2 = 1 - k^2$, and k is defined by the equation

$$K/K' = \log[(U_o/U_i)/(\pi/2 - \alpha)] \tag{5}$$

Among the aforementioned papers giving approximate solutions for the compressible peripheral jet, the most recent,[3] which also contains comparisons of its results with those of the other two and with Strand's incompressible solution,[9] relies on a global horizontal momentum balance and on an approximate velocity profile at the jet exit modeled after an irrotational vortex. This approach, albeit not giving a local representation of the actual velocity field, has the advantage of giving an explicit approximate formula for all of the global parameters of interest.

Present Compressible Solution

We study the fluid-dynamic field in peripheral jets in proximity to the ground, assuming, as in Ref. 9, the region amid the jets to be at rest and the angle θ between the velocity and the x axis (see Fig. 1) to be vanishing at $x = 0$, but we do not assume the density of the fluid to be constant.

The equation that determines the compressible stream function ($\rho^+ u = \psi_y$, $\rho^+ v = -\psi_x$, where u and v are the velocity components in a Cartesian system of coordinates x, y) written in terms of V(modulus of the velocity) and θ, is

$$V^2\left(1 - \frac{\gamma-1}{2}\frac{V^2}{a_s^2}\right)\psi_{VV} + V\left(1 - \frac{\gamma-3}{2}\frac{V^2}{a_s^2}\right)\psi_V + \left(1 - \frac{\gamma+1}{2}\frac{V^2}{a_s^2}\right)\psi_{\theta\theta} = 0 \tag{6}$$

where a_s is the stagnation sound velocity.

To obtain the physical coordinates in terms of V and θ, we may use the following easily derived relations:

$$x_v = \frac{1}{\rho^+ V}(\rho^+\cos\theta\,\varphi_v - \sin\theta\,\psi_v) \tag{7a}$$

$$y_v = \frac{1}{\rho^+ V}(\rho^+\sin\theta\,\varphi_v + \cos\theta\,\psi_v) \tag{7b}$$

$$x_\theta = \frac{1}{\rho^+ V}(\rho^+\cos\theta\,\varphi_\theta - \sin\theta\,\psi_\theta) \tag{8a}$$

$$y_\theta = \frac{1}{\rho^+ V}(\rho^+\sin\theta\,\varphi_\theta + \cos\theta\,\psi_\theta) \tag{8b}$$

$$\varphi_v = (1/\rho^+)_v\,\psi_\theta - \frac{1}{V\rho^+}\psi_\theta \tag{9a}$$

$$\varphi_\theta = V\,\psi_v/\rho^+ \tag{9b}$$

The physical boundary conditions of the problem are (see Fig. 1):

$$V = \text{const} = V_i \qquad \text{over CD} \tag{10}$$

$$\theta = \frac{\pi}{2} + \alpha \qquad \text{over DA}_o' \tag{11}$$

$$V = \text{const} = V_o \qquad \text{over } BA_o \tag{12}$$

$$\theta = 0 \qquad \text{over CB} \tag{13}$$

the latter being similar to the condition imposed in Strand's incompressible solution. In the hodograph plane, the corresponding boundary conditions for the stream function $\psi(V, \theta)$, also taking into account Eqs. (7-9), turn out to be

$$\psi = 0 \qquad \text{over CD} \tag{14}$$

$$\psi = 0 \qquad \text{over DA}_o' \tag{15}$$

$$\psi = q = \rho_o^+ V_o t_o \qquad \text{over } BA_o \tag{16}$$

$$\psi_\theta = 0 \qquad \text{over CB} \tag{17}$$

In particular, Eq. (17) is obtained after observing that, being the segment CB, where $\theta = 0$, equipotential, there $\varphi_v = 0$; from Eq. (9a), Eq. (17) then follows.

In Eqs. (6-17) subscripts o and s denote room and stagnation conditions, respectively, and t_o is the jet thickness at infinity.

The solution of Eqs. (6) and (14-17) can be obtained by expressing ψ by means of a Fourier series in the following form:

$$\psi = \sum_1^\infty a_n K_n(\xi) \sin[(\pi/2 - \theta^*)(2n - 1)] \tag{18}$$

where

$$\xi = \frac{V}{V_i}, \qquad \theta^* = r\theta \left(0 \le \theta^* \le \frac{\pi}{2}\right), \qquad r = \frac{\pi/2}{\pi/2 + \alpha} \tag{19}$$

and $K_n(\xi)$ and a_n are unknown functions and coefficients.

Equation (18) satisfies Eq. (17) identically; by putting

$$K_n(1) = 0 \tag{20}$$

Eq. (14) and Eq. (15) are also satisfied. By expressing the constant q by means of a Fourier series as

$$q = (4q/\pi) \sum_1^\infty \sin[(\pi/2 - \theta^*)(2n - 1)]/(2n - 1) \tag{21}$$

Eq. (16) is satisfied if

$$a_n = 4q/[\pi(2n - 1)\, K_n(\xi_o)] \qquad (\xi_o = V_o/V_i) \tag{22}$$

The $K_n(\xi)$ is given by the equation

$$\xi^2\left(1 - \frac{\gamma - 1}{2}\frac{V^2}{a_s^2}\right)K_n'' + \xi\left(1 - \frac{\gamma - 3}{2}\frac{V^2}{a_s^2}\right)K_n'$$
$$- \left(1 - \frac{\gamma + 1}{2}\frac{V^2}{a_s^2}\right) r^2 (2n - 1)^2 K_n = 0 \tag{23}$$

solved with the condition (20) and

$$K_n'(1) = 1 \tag{24}$$

In the incompressible case ($a_s \to \infty$), Eqs. (23), (20), and (24) give

$$K_n(\xi) = [\xi^{r(2n-1)} - \xi^{-r(2n-1)}]/[r(4n - 2)] \tag{25}$$

Chaplygin found an analytical expression of the solutions of Eq. (23) in terms of hypergeometric functions. In applying this expression, numerical problems are encountered when the hypergeometric functions must be evaluated from their power series for large n, so that new methods of evaluation are still being studied today.[11] We decided to bypass these problems and solve Eq. (23) numerically by a standard Runge-Kutta algorithm, which can always be done with ease. To obtain sufficient precision, we adopted a discretization varying from 40 steps for low n and small V_o/V_i up to 200 steps for $n = 40$ and $V_o/V_i = 25$.

Formulas for the Evaluation of the Performance of the Hovercraft in the Compressible Regime

To analyze the fluid-dynamic field generated by the peripheral jets of the hovercraft we need the characteristics of the flow at $x = 0$ (see Fig. 1). These properties can be given in different ways, but if the ambient pressure is known, the most important data are the total enthalpy (or stagnation sound velocity), the flow rate, the direction of the stream, and the base pressure (or the velocity on the inner streamline).

Therefore we shall assume to have been given the following data: V_o/a_s (velocity on outer streamline/stagnation velocity of sound), V_o/V_i (V_i is the velocity on the inner streamline), $q = V_o t_o\, \rho_o/\rho_s$, and α (angle of the jet with respect to the normal to the ground at $x = 0$).

Inserting these data into the procedure just explained leads one to determine the stream function field in the form of the series (18). Hence we can obtain the various parameters and, in particular, the position of point D (Fig. 1) where the jet attaches to the ground.

When $\xi = \text{const}$, by integrating Eq. (8a), one has

$$2r\rho^+ x(\theta, \xi) = \sum_1^\infty a_n K_n'(\xi)(p_n/c_n + s_n/d_n)$$
$$+ (r/\xi) \sum_1^\infty a_n K_n(\xi)(2n - 1)(p_n/c_n + s_n/d_n)$$

where

$$c_n = 2n - 1 - 1/r, \qquad d_n = 2n - 1 + 1/r,$$

$$p_n = \cos[c_n\theta^* - (2n - 1)\pi/2] \qquad s_n = \cos[d_n\theta^* - (2n - 1)\pi/2]$$

In particular for the position of point D ($\theta^* = \pi/2, V = V_i$) where the jet attaches to the ground (Fig. 1), it results that for $r = 1$

$$x(\pi/2, 1)/t_o = \xi_o\, \rho(\xi_o)/\rho(1)\, K_1(\xi_o) \tag{26}$$

and for $r < 1$

$$x(\pi/2, 1)/t_o = \frac{2\xi_o\rho(\xi_o)}{r\pi\rho(1)} \cos(\pi/2r) \sum_1^\infty \frac{(1/c_n + 1/d_n)}{(2n - 1)K_n(\xi_o)} \tag{27}$$

In a similar way from Eq. (8b) one has

$$y(\pi/2, 1)/t_o = \frac{2\xi_o\rho(\xi_o)}{r\pi\rho(1)}$$
$$\times \sum_1^\infty \frac{\sin(\pi/2r)(1/c_n + 1/d_n) + (-1)^n(1/c_n - 1/d_n)}{(2n - 1)K_n(\xi_o)} \tag{28}$$

where $1/c_1 = 0$ for $r = 1$, and

$$h/t_o = x(\pi/3, 2, 1)\cos\alpha/t_o + [y(\pi/2, 1)/t_o - t_e/t_o]\sin\alpha \tag{29}$$

By integrating Eq. (7b) for $\theta = \text{const}$ one has

$$t_e/t_o = y(0, \xi_o)/t_o = -\frac{4\xi_o\rho^+(\xi_o)}{\pi} \int_1^{\xi_o} \frac{\rho^+\xi}{\xi}$$
$$\times \sum_1^\infty \frac{K_n'(\xi)}{K_n(\xi_o)} \frac{(-1)^n}{(2n - 1)}\, d\xi \tag{30}$$

Equations (29) and (30) give the ratio t_e/h as a function of V_o/a_s, ξ_o, and α.

Once the coordinates of the point D are known, Eq. (29), together with Eqs. (27) and (28), enables us to obtain h/t_o, the ratio between height from ground and thickness of the jet at infinity; Eq. (30) gives t_e/t_o ($= y_B/t_o$), the ratio between the thicknesses of the jet at the exit and at infinity. Thus, one has t_e/h [$= (t_e/t_o)/(h/t_o)$], in terms of the data of the problem.

The pressure coefficient $c_p = (p - p_o)/(\frac{1}{2}\rho_o V_o^2)$ is given in the incompressible and compressible cases, $c_{p,\text{inc.}}$ and $c_{p,\text{com.}}$ as follows:

$$c_{p,\text{inc}} = 1 - \xi^2 \xi_o^{-2} \tag{31}$$

$$c_{p,\text{com}} = \frac{p^+ - p_o^+}{\frac{1}{2}\gamma p^+(\xi_o)\, V_o^2/a_s^2} \tag{32}$$

The nondimensional pressure p^+ vs M is given by

$$p^+ = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\gamma/(\gamma - 1)} \tag{33}$$

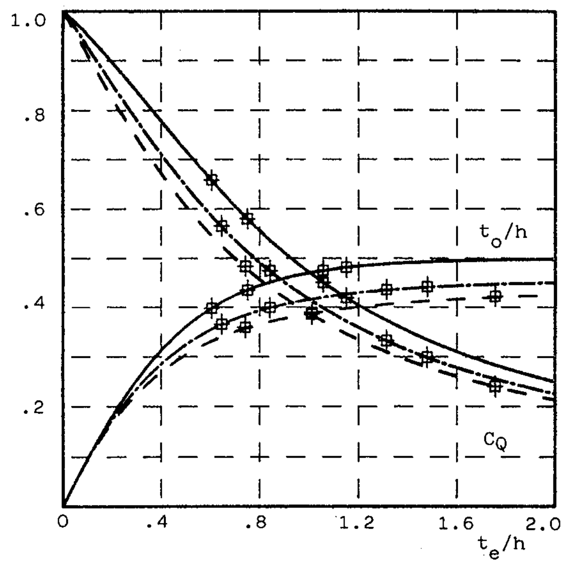

Fig. 2 Jet contraction ratio t_o/h and mass-flow coefficient C_Q vs thickness/height ratio t_e/h for $M_o = 0$ and jet exit angle $\alpha = 0$, 30 and 60 deg: ——, $\alpha = 0$ deg; — · —, $\alpha = 30$ deg; — —, $\alpha = 60$ deg; ⊕ from Ref. 9.

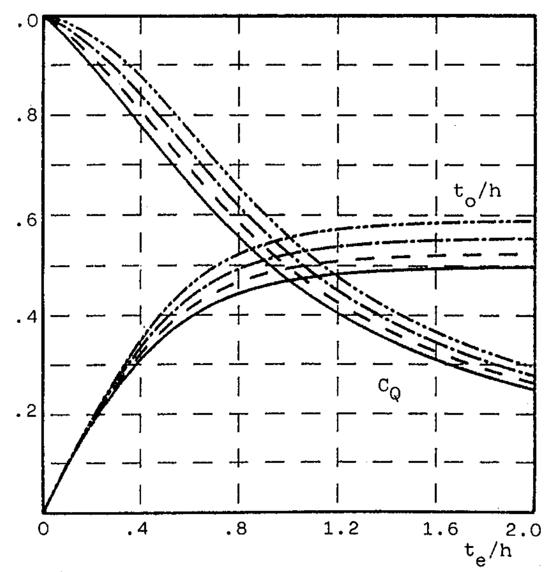

Fig. 3 Jet contraction ratio t_o/h and mass-flow coefficient C_Q vs thickness/height ratio t_e/h for jet exit angle $\alpha = 0$ deg and $M_o = 0$, 0.46, 0.66, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

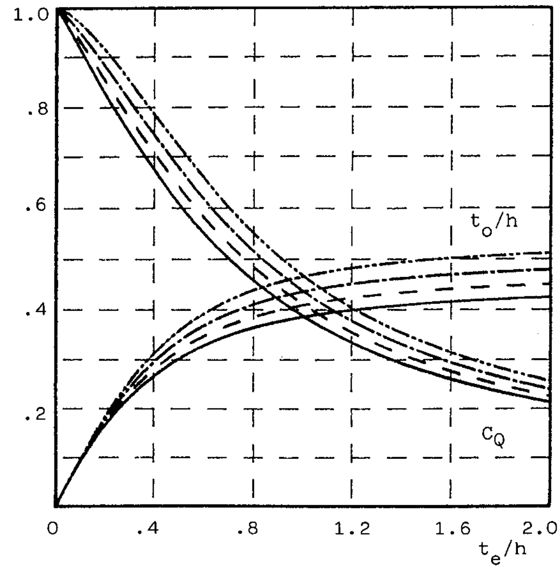

Fig. 4 Jet contraction ratio t_o/h and mass-flow coefficient C_Q vs thickness/height ratio t_e/h for jet exit angle $\alpha = 60$ deg and $M_o = 0$, 0.46, 0.66, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

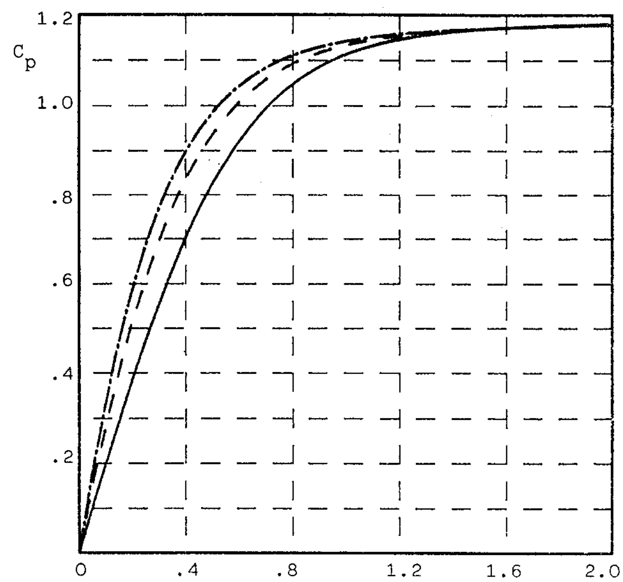

Fig. 5 Pressure coefficient C_p vs thickness/height ratio t_e/h for $M_o = 0.83$ and jet exit angle $\alpha = 0$, 30, and 60 deg: ——, $\alpha = 0$ deg; — —, $\alpha = 30$ deg; — · —, $\alpha = 60$ deg.

and M^2, as a function of ξ, is

$$M^2 = \frac{(V_o/a_s)^2}{\xi_o^2/\xi^2 - (V_o/a_s)^2(\gamma - 1)/2} \tag{34}$$

The mass-flow coefficient, defined as $C_Q = Q/\rho_o V_o t_e$ where $Q = \rho_o V_o t_o$, turns out to be given by

$$C_Q = t_o/t_e \tag{35}$$

[which may be obtained from Eq. (30)].

The momentum jet coefficient is

$$C_j = 2J/\rho_o V_o^2 t_e \tag{36}$$

where

$$J = \int_o^{t_e} [p - p_o + \rho V^2]\, \mathrm{d}y \tag{37}$$

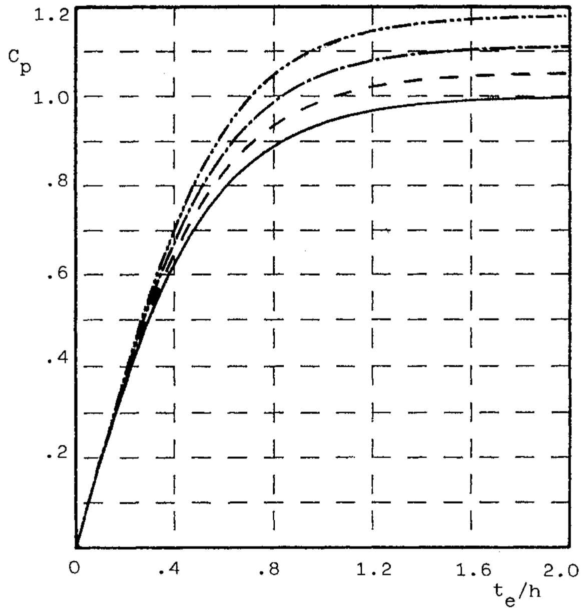

Fig. 6 Pressure coefficient C_p vs thickness/height ratio t_e/h for jet exit angle $\alpha = 0$ deg and $M_o = 0, 0.46, 0.66$, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

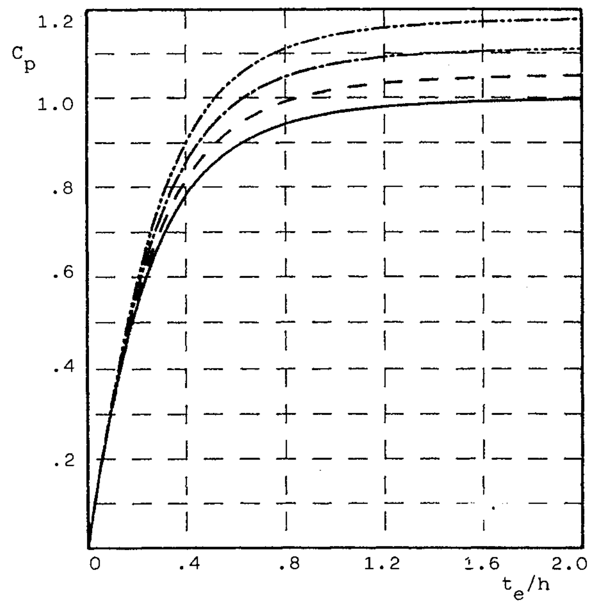

Fig. 7 Pressure coefficient C_p vs thickness/height ratio t_e/h for jet exit angle $\alpha = 60$ deg and $M_o = 0, 0.46, 0.66$, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

We have the following expressions for the incompressible and compressible cases:

$$C_{j,\mathrm{inc}} = (t_o/t_e)\int_o^{t_e/t_o} (c_{p,\mathrm{inc}} + 2\xi^2/\xi_o^2)\mathrm{d}y/t_o \tag{38}$$

$$C_{j,\mathrm{com}} = (t_o/t_e)\int_o^{t_e/t_o} \left[c_{p,\mathrm{com}} + \frac{2\rho(\xi)}{\rho(\xi_o)\xi_o^2}\right] \mathrm{d}y/t_o \tag{39}$$

The lift coefficient C_L is given by

$$C_L = 2L/\rho_o V_o^2 l_T) \tag{40}$$

where

$$L = 2\,J\cos\alpha + (p_i - p_o)l \tag{41}$$

$$l_T = l + 2t_e\cos\alpha \tag{42}$$

It results that, with $K_e = t_e/l_T$,

$$C_L = 2C_j K_e \cos\alpha + C_p(l - 2K_e\cos\alpha) \tag{43}$$

where $C_p = 2(p_i - p_o)/\rho_o V_o^2$.

For the purpose of comparing our results with those of Ref. 3, which also reports the results of Refs. 1, 2, and 9, we may introduce a power coefficient according to the definition

$$C_w = \frac{\tfrac{1}{2}\rho_o t_o V_o^3}{(2/\rho_o)^{1/2}(p_i - p_o)^{3/2}h} = \frac{1}{C_p^{3/2}}\frac{t_o}{h} \tag{44}$$

where the numerator represents the kinetic-energy flow in the jet and the denominator the power that would be expended, if the fluid were incompressible, to obtain the same p_i and h from an ideal static plenum chamber.

This coefficient may in practice be used to determine the optimal jet thickness t_e for a given Mach number M_o, exit angle α, base pressure, and height from ground. (If the purpose were to determine the variation of expended power with height for a given geometry, and thus jet thickness t_e, the second power coefficient defined in Ref. 3 would be more useful. A plot of this second coefficient, as for instance in Fig. 7 of Ref. 3, appears as a regularly increasing curve that has, contrary to the plots of C_L and C_w, no maxima or minima. Therefore, as one should expect, for a given machine expended power increases monotonically with height from ground.)

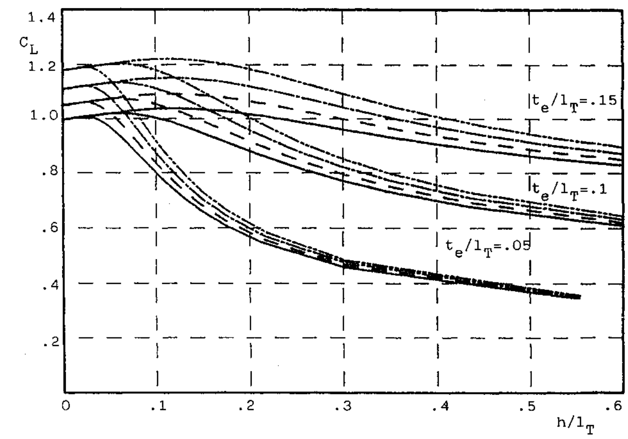

Fig. 8 Lift coefficient C_L vs height/width ratio h/l_T for $\alpha = 0$ deg and $M_o = 0, 0.46, 0.66$, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

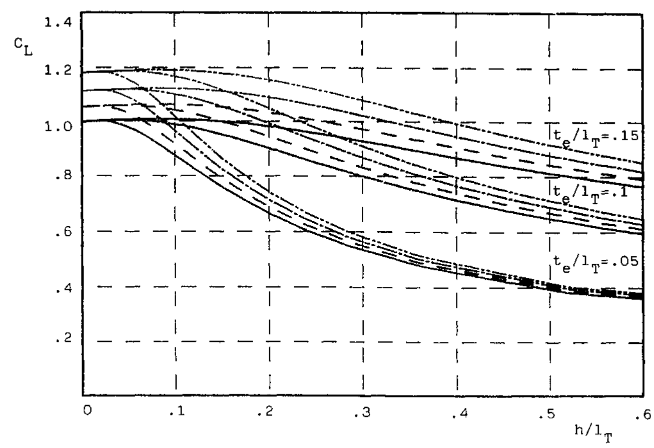

Fig. 9 Lift coefficient C_L vs height/width ratio h/l_T for $\alpha = 60$ deg and $M_o = 0, 0.46, 0.66$, and 0.83: ——, $M_o = 0$; — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$.

Analysis of the Results

Figure 2 plots the mass-flow coefficient of Eq. (35) and the ratio t_o/h vs t_e/h, for $\alpha = 0$, 30, and 60 deg in the incompressible case ($V_o/a_s = 0$), as obtained by the present method and by Strand's; the corresponding curves practically coincide.

Figures 3 and 4 show the corresponding curves for $M_o = 0$ (incompressible case), 0.46, 0.66, and 0.83 for $\alpha = 0$ and 60 deg; one can see that for any t_e/h, and in particular for $t_e/h \to \infty$, both ratios t_o/t_e (i.e., C_Q) and t_o/h increase as M_o increases; moreover, for given t_e and h the jet thickness at infinity increases as M_o increases.

Figures 5-7 show the curves of the pressure coefficient C_p [$= 2(p_i - p_o)/\rho_o V_o^2$] vs t_e/h; in particular in Fig. 5 there are the cases of $\alpha = 0$, 30, and 60 deg for $M_o = 0.83$, and in Figs. 6 and 7 there are the cases of $M_o = 0$, 0.46, 0.66, and 0.83 for $\alpha = 0$ and 60 deg. One can see that C_p increases as t_e/h increases, and its asymptotic value (for $t_e/h \to \infty$) is independent of α and increases with M_o.

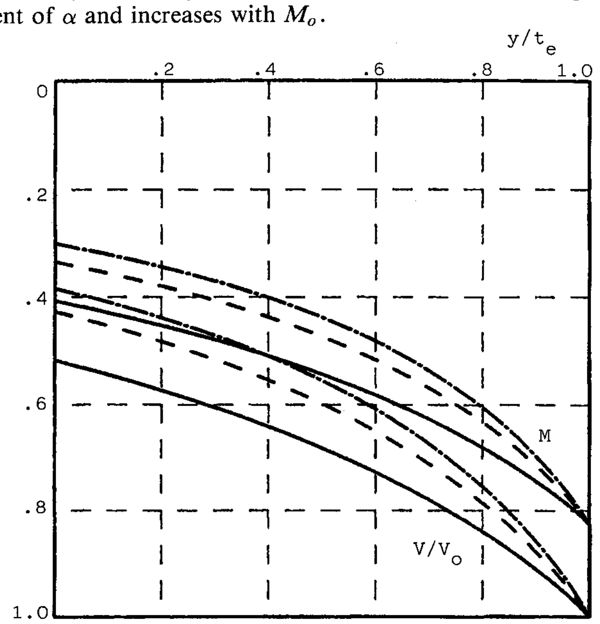

Fig. 10 Nondimensional velocity V/V_o and Mach number distribution across jet exit vs nondimensional coordinate y/t_e for $t_e/h = 0.5$, $M_o = 0.83$, and jet exit angle $\alpha = 0$, 30, and 60 deg: ——, $\alpha = 0$ deg; — —, $\alpha = 30$ deg; — · —, $\alpha = 60$ deg.

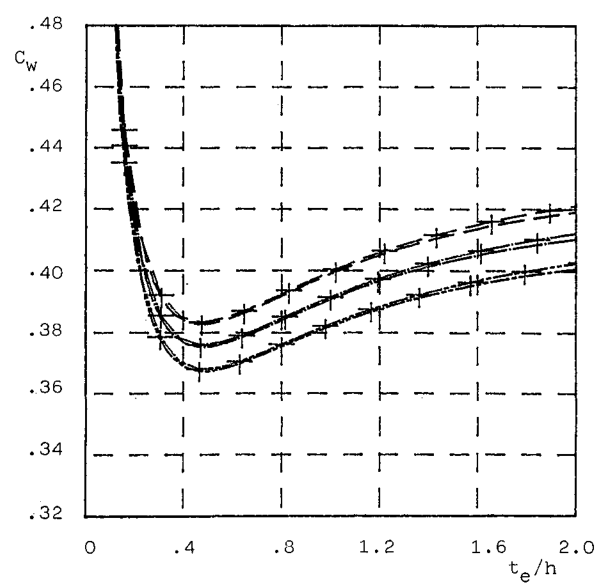

Fig. 11 Power coefficient C_w vs thickness/height ratio t_e/h for $\alpha = 60$ deg and $M_o = 0.46$, 0.66, and 0.83: — —, $M_o = 0.46$; — · —, $M_o = 0.66$; — ·· —, $M_o = 0.83$; + from Ref. 3.

In Figs. 8 and 9 there are the curves of the lift coefficient C_L ($= 2L/\rho_o V_o^2 l_T$) vs h/l_T (distance from ground/total width of the machine) for $M_o = 0$, 0.46, 0.66, and 0.83 and for $t_e/l_T = 0.05$, 0.1, and 0.15. One can see that C_L increases with M_o and t_e/l_T; in particular for $M_o = 0.83$, $C_L/C_{L,\text{inc}} = 1.3$.

When $h/l_T \to 0$, $C_L \to C_p$.

Figure 10 shows the velocity and Mach number profiles at $x = 0$ for $t_e/h = 0.5$ and $M_o = 0.83$ for $\alpha = 0$, 30, and 60 deg.

Figure 11 shows the dependence of the power coefficient of Eq. (44) on the ratio t_e/h. The same figure also contains the corresponding curves calculated from approximate formulas of Ref. 3. The agreement may be seen to be very good for mid to low values of the ratio t_e/h, which are the values of greatest practical importance. It is interesting to observe that for $\alpha = 60$ deg the optimal value of t_e/h turns up nearly independent of M_o and is about 0.5.

These data, just as Hope-Gill's, refer to an infinite value of the ratio l_T/h. We have also considered finite values of this ratio, finding that the power coefficient decreases even further.

Concluding Remarks

In this paper we presented an exact solution of the compressible inviscid two-dimensional fluid-dynamic equations that govern the flow in the air cushion determined by peripheral subsonic jet in a ground-effect machine: the interest of such a study arises from the construction of high-performance hovercraft planned in both commercial and particular services.

This solution was obtained by expressing the stream function by means of a suitable Fourier series in the hodograph plane and gives, in the incompressible regime, the same results as the conformal mapping theory.

All of the quantities of interest were calculated and compared with the corresponding incompressible results of Ref. 9 for several values of the outer Mach number and exit angle of the jet. This comparison shows that the influence of the compressibility is at most on the order of magnitude of 20% for high subsonic Mach numbers. In addition, our results confirm the validity of the approximate solution given in Ref. 3, with which they agree.

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